

COMPARISON OF REAL TIME IMPACT LOAD IDENTIFICATION PROCEDURES

Krzysztof Sekula^{*}, Jan Holnicki-Szulc^{*}

^{*} Smart-Tech Centre,
Institute of Fundamental Technological Research,
Świętokrzyska 21 00-049 Warsaw,
e-mail: ksekula@ippt.gov.pl
e-mail: holnicki@ippt.gov.pl
<http://smart.ippt.gov.pl/warsaw/>

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***Abstract.** The objective of the paper is to present the current research state, which was performed for searching efficient methods of impact load identification. An important issue is that invented algorithms should be able to perform the identification in real time. General motivation of the performed research is to apply proposed concepts in an adaptive impact absorption system (AIA).*

The paper includes verification of algorithms of the real time load identification invented by the authors. The comparison of them is enclosed as well. A few parameters were considered for verification of the proposed procedures. First of all the operation time was taken into account. It is accounted for in order to verify fulfilling the real time condition. Moreover, the accuracy and simplicity of the system was considered (required number of sensors is analysed). And finally the necessity of fixing of the sensor directly to the impacting body.

First part of the paper includes the general formulation of the proposed algorithms of the load identification. The presentation of the set-up used for development the formulated algorithm was included later on. Finally the experimental verification of algorithms was performed employing a laboratory test. On the basis of the above the comparison of the procedures were attached and conclusions were drawn.

1 INTRODUCTION

Load identification constitutes an important type of engineering problem. It belongs to an inverse problem category because its objective is to determine the reason on the basis of the result. The paper discusses the possibilities of determination of the load acting on the structure on the basis of the measured response. When a static or quasi-static load is considered, its identification is usually easy to perform. However, the level of difficulty increases drastically in the case of dynamic excitation. Many difficulties tend to appear especially when impact loads are being considered. It is caused by short duration of the phenomenon and by the appearance of the relatively high load values. This paper will be focused on the identification in the case of impact load.

Many techniques were developed in order to identify parameters of impact load. In the paper [1] a brief review of methods used in the case of the indirect impact force is presented. The article considers a variety of approaches to measuring the time history of the impact force, its direction and location. The presented methods are mainly based on the deconvolution techniques performed in the time and the frequency domain. The techniques for moving and impact load based on neural networks have also been proposed by some researches [2,3,4]. Some of the papers are devoted to the techniques performed on the basis of the exact analytical solution or applied approaches based on finite element formulation. The comparison of the last two techniques are presented in the article [5]. Most of the methods have been developed and tested with reference to simple continuous structures like beams [6] and plates [7]. Contrary to the reference in this paper the structure which could be modeled as a discrete structure will be considered.

Most of the publications do not discuss the operational time of applied methods. The identification techniques presented in this paper are very strongly focused on minimizing the time required to apply. It is crucial to fulfill the real time condition. The objective of the analysed methods is to be applicable to the AIA and fast enough to predict the parameters of the impact. These quantities could be used for optimal control of the AIA and for smoothing down structural dynamic response due to random impact loads.

2. DISCUSSION OF CHOSEN PARAMETERS OF IMPACT LOAD IDENTIFICATION

The impact load monitoring problem is discussed as identification of the mass and the parameters of the motion of the object impacted into the structure. Fig. 1 schematically represents the mass and velocity effect of the impacted body on the dynamical response of the structure. The group of the curves is determined by the combination of the mass and velocity of body impacted in to the cantilever truss characterized by the same quantity of kinetic energy in the moment of impact. It means that we can determine two main areas in the mass – velocity space. *Fast dynamics* (the area 1 Fig.1) corresponding to small mass impacting with high velocity which causes deformation of the structure close to the impact point. *Slow dynamics* (the area 2 Fig.1) corresponding to heavy mass impacting with slow velocity which causes deformation of the structure close to its support similar to the case of static response.

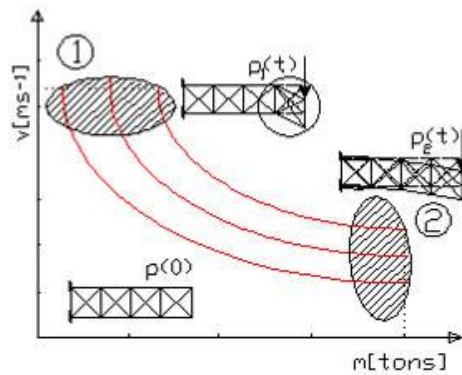


Fig 1. Influence of velocity and the mass of the impacting object on structural response [8]

According to the above, a hypothesis could be proposed: "The minimum information required to determine impact load (in the location of the sensor) consists of two quantities: the initial impact velocity and the dropping mass (or kinetic energy)" [8].

3 SOLUTION MAPS HEURISTIC APPROACH

The proposed methodology of load identification is based on an inverse problem. In general the approach can be determined by inferring the value of a reason from the results. In our case the load parameters are considered the reason and the response of a structure the result.

One of the main difficulties is that inverse problem does not have to lead to a univocal solution [9]. Therefore this approach can be applied in case of well defined systems. The important condition is that the dynamic response of the structure must differ in case of different excitations.

To apply considered method, the solution map should be prepared with its database (storing dynamic responses to different excitation scenarios). It is reasonable to perform the solution map based on sensitivity analysis. The best way is usually to obtain it during experiment but in many cases it is justified to use a computer model if it is reliable. In testing the values with the strongest influence on the response should be considered and used as the variable during research. Usually in case of load identifications problems such parameters are: position and direction of excitation, value of the load and its dynamics and time history. In many cases it is reasonable to neglect some less important variable (which proves to have little influence on the response) like temperature, humidity etc. because of the simplicity of the method.

The proposed approach can be determined as an optimization problem. From the existing solution map the best solution for the considered measurement should be found. It could be obtained by minimization of the difference between actually measured response and a group of saved ones. The number of searching parameters is limited by the number of measured variables.

Measured response of the structure (amplitude of excitation, period etc.) could be denoted by Y , and is dependent on parameters of excitations being sought for instance (mass, velocity, acceleration amplitude etc.) and denoted by x . This can be stated by the formulation (1).

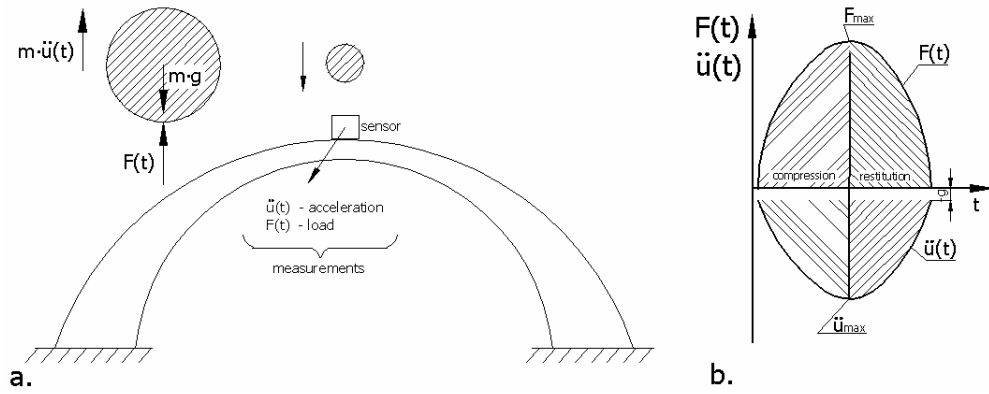


Fig 2. Dynamic approach illustration: a. schema of the structure and a dropped mass, b. measured signals during the contact of the dropping mass with the structure.

Mass of the impacted body can be obtained from the formula (5).

$$m = \frac{F(t)}{g - \ddot{u}} \quad (5)$$

So according to the above if we assume that the impacting body is equipped with a set of two sensors used for the force and acceleration measurements the mass identification will be possible. From a theoretical point of view the mass could be determined from the very beginning of the impact phenomena, so in real time.

Another parameter necessary to identify is velocity. This parameter could be directly measured by applying the additional velocity detector. Nevertheless it is always reasonable to build as simple sensor system as possible and the best way is to use minimum number of detectors. In our case the velocity could be determined indirectly on the basis of the measured acceleration with signal conditioning procedure based on its integration. To apply it the following formula could be proposed (6).

$$\dot{u}(t) = \int_{t_0}^{t_1} \ddot{u}(t) dt \quad (6)$$

The main advantage of the dynamic approach is the simplicity of the procedures used for identification of load parameters (mass, velocity). But on the other hand the disadvantage of the method is the necessity of locating the sensing elements close to the impacted body which could limit its use. The analysis of the method was preliminarily performed in case of the drop testing stand equipped with a magnetorheological damper, see [12]. This paper will discuss the practical laboratory experiment with proposed approach applied, more in-depth consideration will be presented further.

5 EXPERIMENTAL SET-UP

In order to perform the feasibility study of a real time dynamic load identification technique the experimental drop testing stand was used (see Fig. 3). The main parts of the set-up are the pneumatic cylinder (1) mounted in a vertical position, the frame (2) and the carriage (3). The lift mechanism includes an electromagnet (4) used for releasing the dropping mass (5) which is guided by the rail system (6) embedded in the frame. Dropping mass is impacted via rubber bumper (7) onto the pneumatic cylinder. During the tests the following signals were acquired: the force signal from the piezoelectric sensor (8) fixed to the piston rod of the pneumatic cylinder in order to measure the full impact history, the signal from the optical switch (9) acting as a trigger and enabling determination of the horizontal speed of the carriage just before the impact. The test procedure also covered measuring of the acceleration in two points: deceleration of the falling mass (10 a) and acceleration (10 b) of the piston rod of the pneumatic cylinder. Additionally the pressure in the cylinder was measured by the use of the 'fast' pressure sensor (11) and the displacement of the piston by use the LVDT sensor (12).

The data acquisition set-up enabled to obtain real time measurements of all signal used in the experiment. The set up includes the conditioning parts like signal amplifiers as well.

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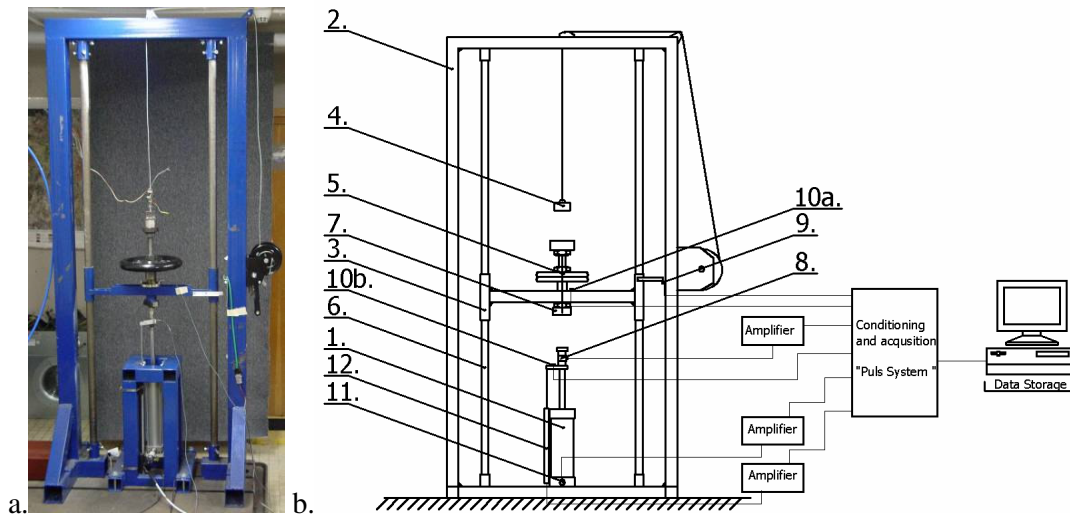


Fig. 3 Experimental testing stand, a. photo of the setup, b. scheme of the setup.

6 CHARACTERISTIC OF THE IMPACT PROCESS

In order to improve the real time identification procedures a thorough analysis and consideration of impact phenomena between the dropping mass and the gas spring (pneumatic cylinder) was performed. This research led to understanding the features of impact phenomenon for this case of structure. In order to obtain it, the sensitivity analysis was

performed. The research concerns the relationship between measured signals of the structure response in terms of impact parameters. The mass of the impactor and the velocity of it in the moment of impact as the variable parameters was used. The described drop-test stand enables us to simulate the wide variety of impact scenarios. The mass range was 7.2-50 [kg] and impact velocity was dependent on the drop height and its range was 0-50 [cm]. As the impact absorption system the gas spring was applied. The pneumatic cylinder had the 63 [mm] of diameter and 250 [mm] of stroke. In the moment of impact the pressure inside the cylinder was atmospheric.

Examples of the measured impacted force are presented in Fig.4. The graphs present the results of the impact force measurements. The results obtained for the impact were characterized by a similar kinetic energy impact approximately equal 30 Nm.

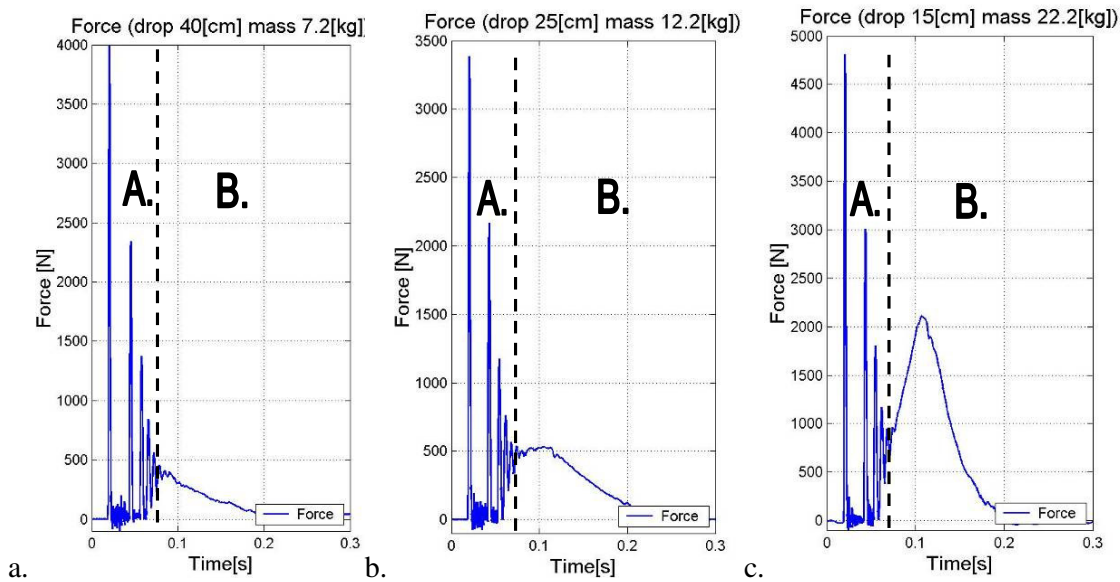


Fig.4 Impact force measurements: a. drop height 40 cm, mass 7.2 kg; b. drop height 25 cm, mass 12.2 kg; c. drop height 15 cm, mass 22.2 kg; A- First impact phase, B- Second impact phase.

The experimental results enable us to draw a number of conclusions concerning the quantitative and the qualitative behavior of impact phenomenon. Some of them are presented below.

For the observed impact phenomenon it is possible to determine two phases of the process as it was shown in Fig.4. The first one is characterized by few rebounds between the falling mass and the piston. The number of them and characteristic reduction of their amplitudes in the successive rebounds and the time period between them is similar for the whole range of boundary conditions (mass, velocity) used in the experiment. The duration of the first impact phase was approximately 50 ms and the duration of an average peak (due to contact) was approximately 5 ms.

The qualitative aspect of the first phase of the phenomena enable to draw the conclusion that the value of the impact force on the beginning of the impact shows rather low sensitivity on the mass value and much stronger on the velocity in the first impact instant.

Moreover, some of the tests were filmed by a high speed camera. Fig.5 shows a sequence of the frames with the 10 ms gap started from the very beginning of the impact. The presented test (Fig. 5) was made for the drop of the 27 kg mass from the 20 cm. The film proved the rebound between the dropped mass and the piston rod.

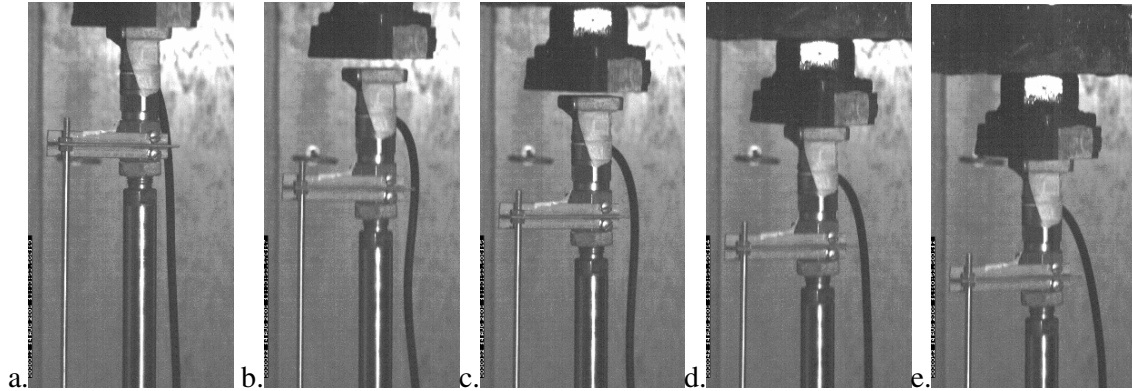


Fig.5 High speed film of impact process: a. 0 [ms], b. 10 [ms], c. 20 [ms], d. 30 [ms], e. 40 [ms].

In the second phase of the impact a simultaneous joint movement of the dropped mass and the piston was observed. In this phase the force value proved much stronger sensitivity on the mass value. The maximum peak of the force value occurred in the maximum compression of the gas spring and it was clearly visible for the higher mass value.

The duration time of the second impact phase was approximately 120-150 ms and the duration of the whole impact phenomenon for the analyzed structure was 170-200 ms.

7 VERIFICATION OF THE IMPACT LOAD IDENTIFICATION PROCEDURES

The approaches proposed in the previous part of the paper were experimentally verified. To perform the tests the experimental setup presented in Part 5 was used. The time histories of measurements of all used sensors were acquired. A wide variety of impact scenarios was used. As the boundary condition seven different masses (7.2, 12.2, 17.2, 22.2, 27.2, 32.2, 37.2 [kg]) were applied. The other parameter was the drop height of the impacting mass. The analysis contained eight different values (5, 10, 15, 20, 25, 30, 35, 40 [cm]), which enables to obtain different values of impact velocities in the range of approx. 1-3[m/s]. Altogether fifty six different impact scenarios were tested.

7.1 Verification of the solution maps approach

The proposed approach was focused on the maximum simplicity of the data accusation set up. The objective was to apply an algorithm able to operate on the basis of measurements from only one sensor. Moreover, an important issue was to use a sensor, which is not directly fixed to the falling mass.

The development of the proposed approach was performed on the basis of signal measured from the force sensor. The researches were focused on identification of the two parameters of the impact (mass and velocity of the impacting body) using the force sensor. It leads to the necessity of determination of more then one quantity on the basis of only one measured force history. In this case it was crucial to chose the signals parameters, which reveal a strong sensitivity on the impact parameters. In Fig.6 an example of force measurement during the impact is presented. The graph shows the chosen parameters (T_u , A_u) used in the load identification procedures based on the solution map approach.

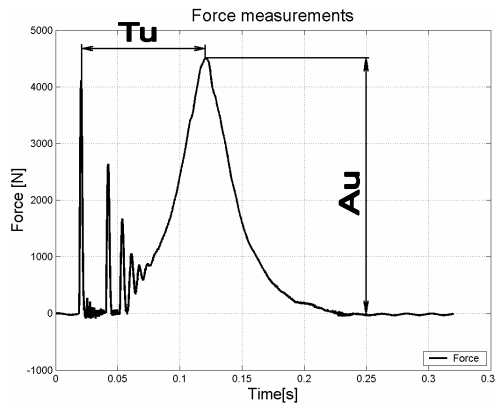


Fig.6 Impact force measurement and the parameters used in the algorithm

To solve the inverse problem the following two parameters obtained on the basis of force measurement were used:

T_u - time period between the maximum force value in the first phase of impact process and the maximum force value measured in second phase of impact process,

A_u - the maximum force value in the second phase of impact process.

The choice of the parameters was motivated by practical aspects. First of all, chosen values must be “characteristic” in the impact process. It means that the parameters should be unambiguously determined in real time during the impact phenomena. Moreover, it was also important that the chosen parameters should prove high sensitivity on the impact parameters (mass and velocity of the impacting body). It is extremely important when the uniqueness of the identification is considered. Fig.7 represents the values of chosen parameters (A_u , T_u) in the mass-velocity “space” .

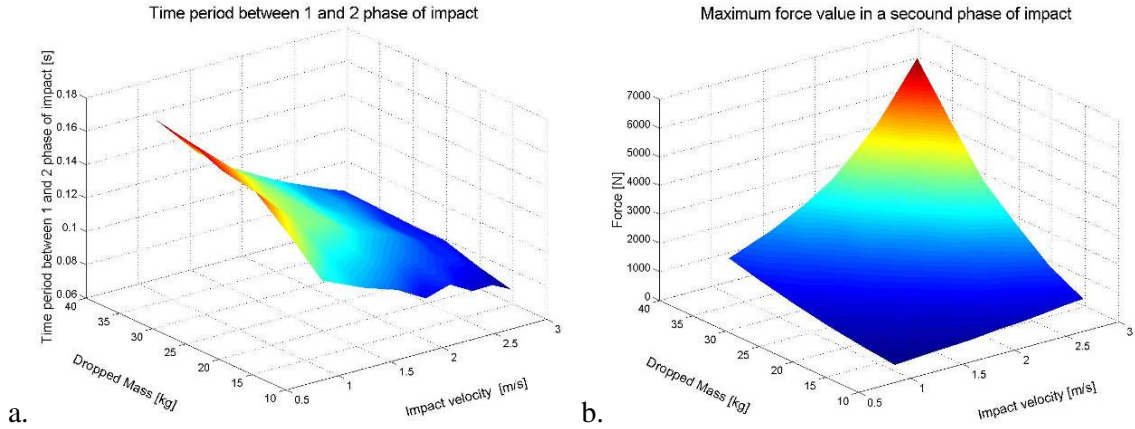


Fig. 7 Sensitivity analysis parameters used in the solution maps approach: a. sensitivity of the parameter T_u in terms of impact quantities (mass, velocity), b. sensitivity of the parameter A_u in terms of impact quantities (mass, velocity).

In the case of the analyzed structure, an identification algorithm proposed on the basis of solution maps approach was formulated. Identification was performed as a problem of finding the minimum of the error function. To achieve it the formula (7) was applied.

$$f\langle m_0, v_0 \rangle = \left[\frac{A_o - A_u(m, v)}{A_o} \right]^2 + \left[\frac{T_o - T_u(m, v)}{T_o} \right]^2 \quad (7)$$

where:

m - mass of the impacting body,

v - velocity of the impacting body in the moment of the impact,

$A_u(m, v)$ - matrix of the values of A_u parameter in terms of mass, velocity, performed on the basis of the sensitivity analysis,

$T_u(m, v)$ - matrix of the values of T_u parameter in terms of mass, velocity, performed on the basis of the sensitivity analysis,

A_o - currently measured value of the A_u parameter,

T_o - currently measured value of the A_u parameter,

m_0, v_0 - Values of the mass and velocity parameters being determined on the basis of measured A_o and T_o .

The identification problem could be presented graphically. Fig.8 presents the experimental results. The contour lines represent constant values of parameters A_u and T_u in terms of mass and velocity of the impacting body.

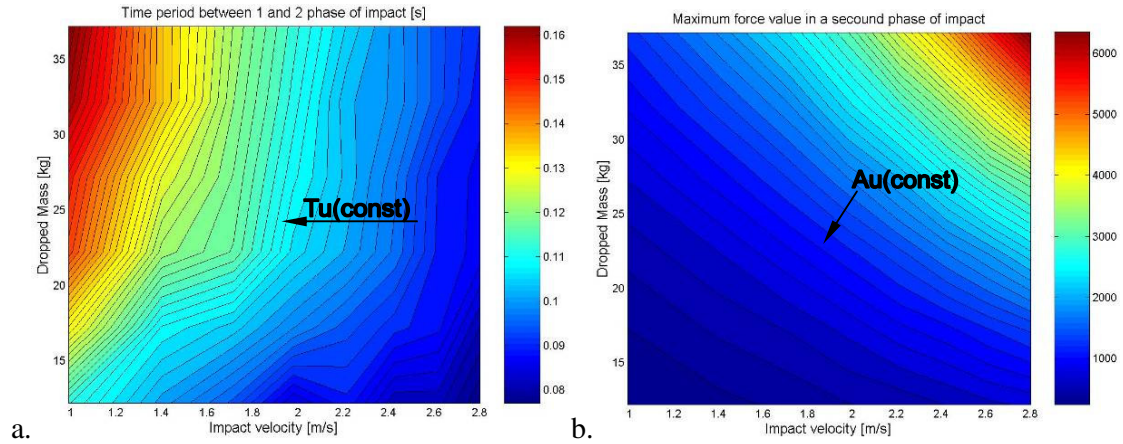


Fig. 8 Sensitivity analysis of parameters used in the solution maps approach: a. sensitivity of the parameter T_u in terms of impact quantities (mass, velocity), b. sensitivity of the parameter A_u in terms of impact quantities (mass, velocity).

The solution maps approach can be explained as the choice of the contour lines which corresponds to the actually measured values of the parameters T_u and A_u . The values being searched for (mass, velocity) could be determined as the identification of the intersection point of the two contour lines as it was schematically shown in Fig.9. The coordinate of the point represents the identified values of the mass and the velocity parameters.

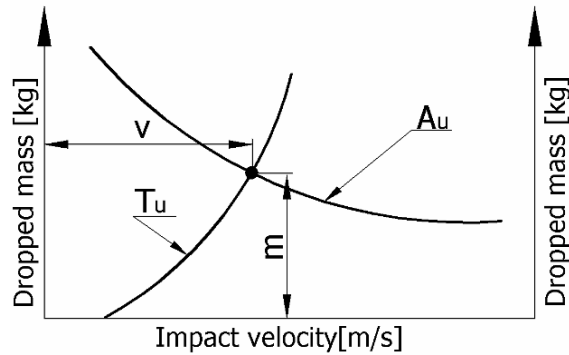


Fig.9 Graphical interpretation of solutions maps approach

A practical verification of this approach was performed in two stages, both required measurements of the force history. The first one was meant to build the solution maps (see Fig. 7 and 8). During the second, the measurements were repeated within the same boundary conditions (in the mass velocity space) as in the first one. The objective was to verify the accuracy of the method. The results are presented in Fig.10. The graph compares the accurate and the identified values of mass (Fig. 10 a) and impact velocity (Fig. 10 b) of the object.

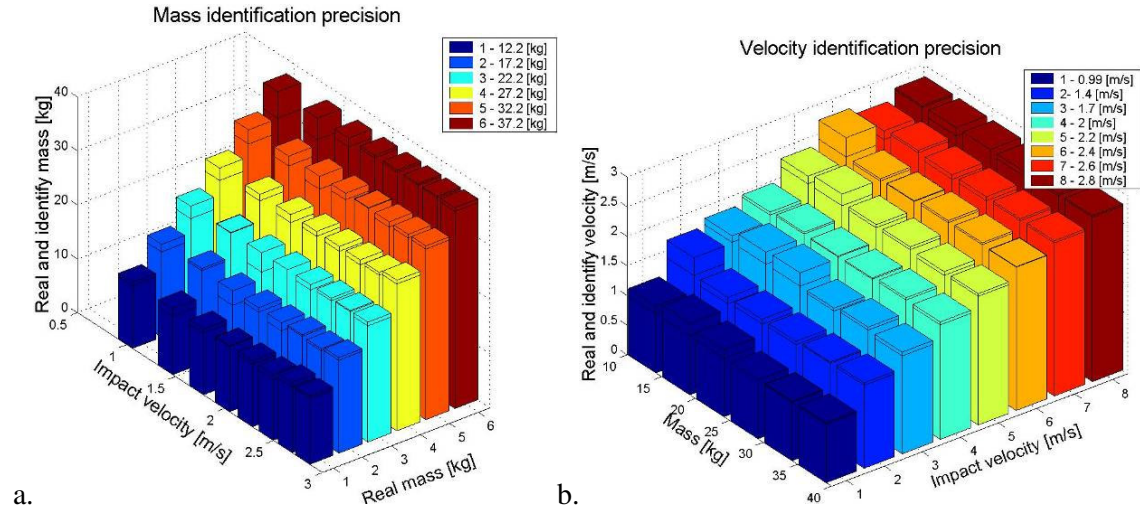


Fig.10 Precision of the parameter identification on the basis of solution maps approach:
 a. mass identification, b. velocity identification.

The precision of parameter identification using a measure based on standard deviation was additionally calculated. Formula 8 was applied. For each group of impact scenarios defined by a constant value of impact velocity or mass the standard deviation was calculated. The constant velocity grouping was used in order to identify the precision of velocity identification. Similarly, groups of constant masses were used for estimating the precision of mass identification. Obtained results are presented in Fig. 11.

$$\sigma = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^n (x_i - \bar{x})^2} \quad (8)$$

where:

- σ - standard deviation,
- n- number of elements in the sample,
- x_i - identified value,
- \bar{x} - exact value.

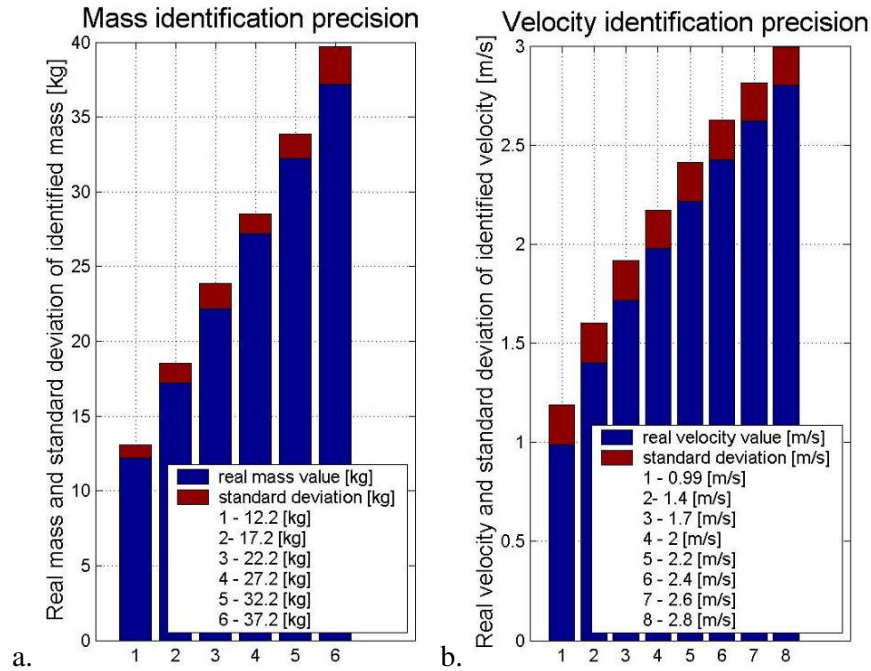


Fig.11 Standard deviation and precise value of the identified parameters: a. mass value, b. velocity value.

To summarize the proposed approach, its advantages and disadvantages can be listed. The important feature is the possibility of application in a relatively wide range of technical systems. Moreover, an important factor is its simplicity. It is especially true if we consider the data acquisition setup, as in the proposed system the approach requires only one sensor. Another positive feature of the method is that the algorithm allows relatively precise determination of the parameters. In case of the analyzed process, the average precision of the determined parameters was $\pm 5\%$. Nevertheless, it should be emphasized that the analyzed process is repetitive, which is a crucial condition.

Nevertheless, the proposed approach has some disadvantages. The concept is limited to structures, which are well defined. Moreover, the application requires performing the tests to determine the solution maps. A crucial disadvantage is a relatively long duration time needed to identify the parameters. In the presented system mass and velocity of the impacting body could be obtained in the second phase of the impact only. The identification was possible first after 100-110 ms from the very beginning of impact process. Moreover, the parameters could be determined only after the maximum peak of the measured force occurred in the second impact phase.

7.2 Verification of the dynamical approach

The objective of the proposed approach is to minimize the time required to determine the impact parameters (mass, velocity). The important issue was to identify the parameters (if possible) on the basis of measurement from sensor not directly fixed to the impacting body.

In the proposed approach for the mass identification Formula (5) was used. It should be emphasized that the equation is fulfilled in the case when impacted body is affecting the structure. Nevertheless, the methodology requires measurements from two sensors: force and acceleration.

The process of mass identification was tested in two cases: on the basis of acceleration of the impacted body and of the piston rod. Accelerations measured in the two locations and the force measurement were characterised by curves, example are presented in Fig 12. The presented results have been measured in the first phase of the impact. They show the very first impulse due to the first contact between the falling mass and the piston rod.

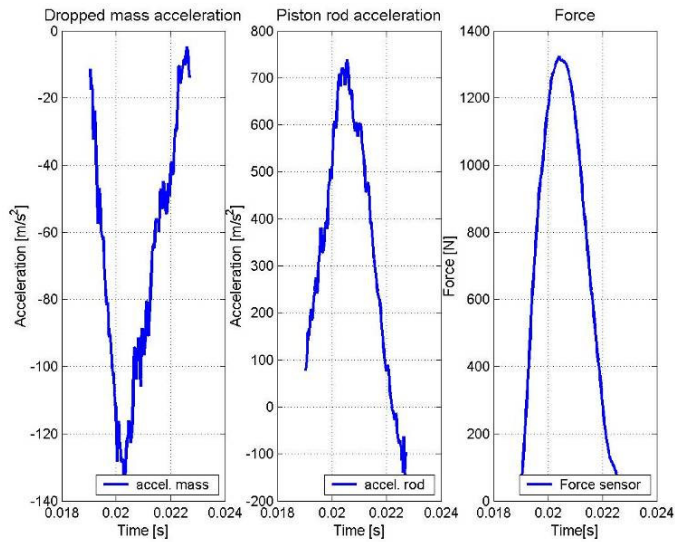


Fig.12 Example of measured signals in the first impulse

The procedure was tested for a wide variety of values of mass of impacted body used in the experiments. Chosen results of identification in the first phase of impact are presented in Fig.13. It could be noted that the obtained mass values have very unstable behaviour. It is mainly caused by the inherent features of the accelerometer sensors. Detectors of this kind are very sensitive to external factors like e.g. vibration of the testing stand caused by the impacts. The graphs contain temporary values of the identified mass, its mean value in a chosen time range. The actual value of mass used in the experiments is presented as well.

The results obtained on the basis of piston rod acceleration (see Fig.13a) show that the identified mass is almost independent on the actual mass and velocity values. This is caused

by the fact that the first phase of the impact was considered. The falling body and the piston rod did not move jointly and rebounds were observed. In general, it is the mass of the piston rod, which is identified in this phase of the phenomena, when the piston rod accelerations are measured.

Much better results of the mass identification (see Fig.13b) were achieved in the case when the deceleration of the falling body was measured. The data enabled to perform reliable dynamic mass identification at the beginning of the impact phenomena. A reliable mass value could be obtained already after 5 ms from the very beginning of the impact phenomenon. The applied methodology enable to obtain the mass value with approximately $\pm 5[\%]$ of accuracy.

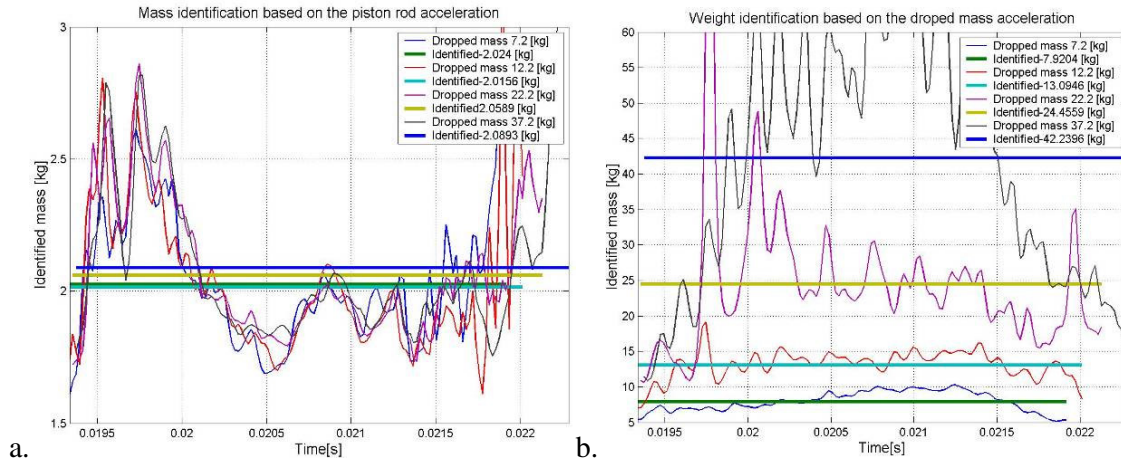


Fig. 13. Mass identification in the first phase of the impact a. acceleration measured on the piston rod of the gas spring , b. acceleration measured on the falling mass

The mass identification on the basis of dynamic approach has been also applied to the signals measured in the second phase of the impact. An example of measurements is shown in Fig.14.

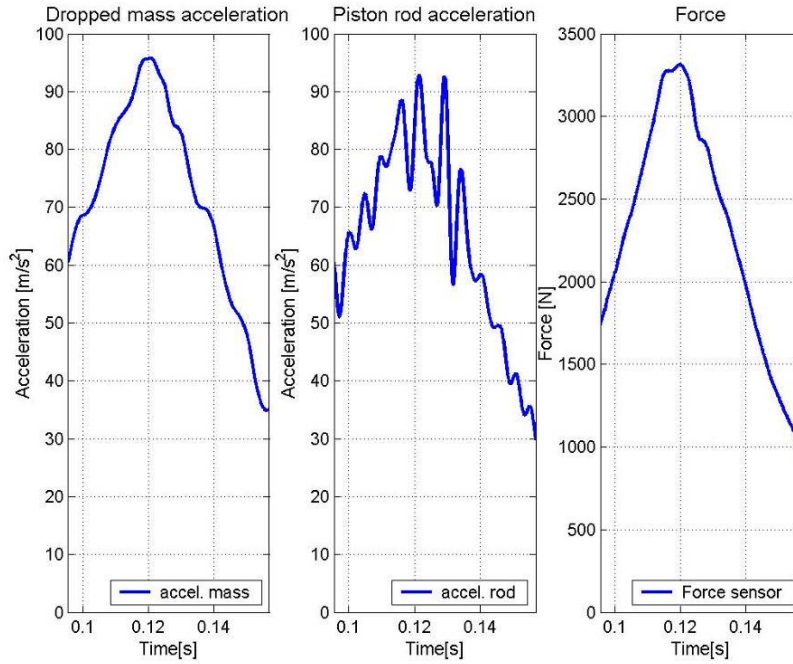


Fig.14 Example of measured signals in the second phase of impact

In the second phase of the impact the common joint movement of the piston rod and the falling mass is observed. It is confirmed by the measurements and a comparison of the acceleration measured on the piston rod and on the falling mass (see Fig. 14).

The mass identification results for the second phase of the impact are presented in Fig.15.

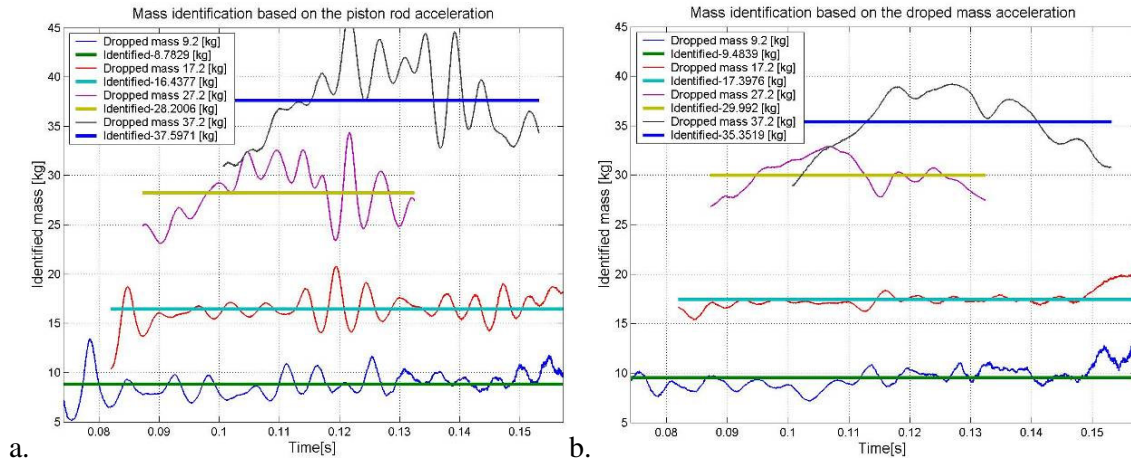


Fig. 14. Mass identification in the second phase of the impact a. accelerated measured on the piston rod of the gas spring , b. accelerated measured on the falling mass.

The mass identification of the impacted body when the second impact phase is considered is feasible. The location of the accelerometer does not have a crucial importance. The values obtained on the basis of both locations reveal relatively similar values. Slightly better results (more stable) were obtained on the basis of acceleration measured on the falling mass. Nevertheless, the values determined from the piston rod are acceptable as well. The accuracy of mass identification for the second impact phase was approximately equal in most of the tested cases $\pm 5[\%]$.

The feasibility study of indirect velocity identification in case of the considered structure was performed as well. Procedures based on signal processing were applied. The measurements of parameters of the piston rod motion were used in the analysis, as the equipment of the stand enable to perform it. Directly measured values like acceleration or displacement have been compared to the signals after conditioning. The obtained results are presented in Fig. 15 a.b.c..

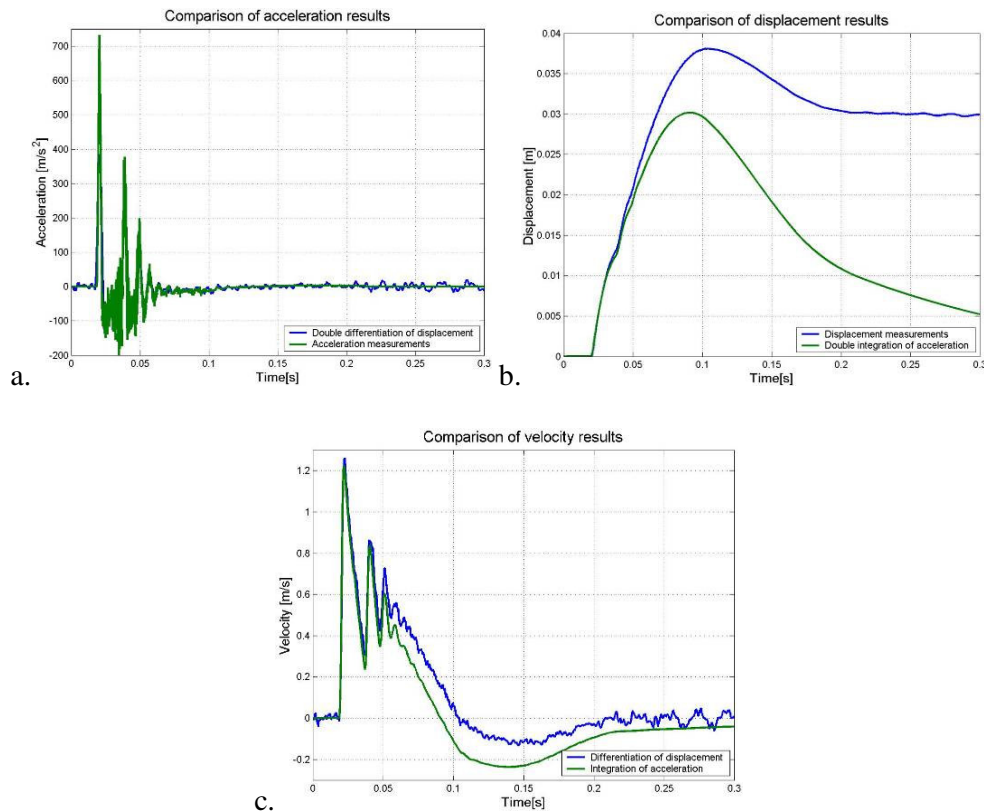


Fig. 15. Direct and indirect measurements motion parameters:

- direct acceleration measured on the piston rod and double differentiation of its displacement,
- direct displacement measured on the piston rod and double integration of its acceleration,
- differentiation displacement measured on the piston rod and integration of its acceleration.

The indirect velocity identification based on the signal processing methods is feasible. Relatively accurate values of the parameters of motion were obtained after the differentiation and integration of the signals. It was proved by the comparison of the quantities measured directly and indirectly (see Fig. 15.a and Fig. 15.c). The velocity could be obtained indirectly by means of differentiation of the displacement signal and by integration of the acceleration, and it is possible to perform this procedure in real time. Nevertheless, initial condition of the velocity is required. In case of the piston rod velocity, the measurements are easy to perform, because its pre impact velocity is zero. More problems in the case of the falling mass velocity identification appear. When acceleration measurements are used, for precise impact velocity identification the whole history of movement is required, which is often difficult to obtain. So, from the practical point of view, the impacting mass identification seems to be feasible when joint movement of the body and the piston are observed. In case of the analyzed structure it was feasible after 80 [ms] from the very beginning of the impact process .

Moreover, the comparison of the velocity identification was performed. Two methods were compared. The first one was performed on the basis of displacement differentiation. The second one involved integration of the acceleration. Both methods led to similar results. Nevertheless, the conditioning of the displacement is more sophisticated and filtering of the signal is usually necessary. It is caused by the fact that the differentiation usually increases the noise effect in contrary to the integration, which smooths it down. For the analyzed structure the conditioning of the acceleration is more justified, especially if the simplicity of the data acquisition set-up is considered.

8 CONCLUSIONS

Real time mass and velocity estimation in case of impact load excitation with only one sensor is extremely difficult. Much better results were obtained when two sensors were applied. For the impact process good results were obtained in case of the combination of an accelerometer and a force sensor. However, the location of the accelerometer is very important. When the accelerometer is fixed to the falling body, precise mass identification is feasible in a short time after the impact. But when the accelerometer is placed on the impacted body, the detection is far more difficult and feasible after a longer period. Therefore, mass determination of the impacting body is possible in short time and with high accuracy only when one of the sensors is fixed to the impacting object or when the parameters of its movement are directly measured. In case of velocity identification on the basis of acceleration measurements the initial condition is needed. In case of the analysed structure the velocity identification was feasible when a joint movement of the piston rod and the falling mass was observed.

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